NAME: KEY

Astronomy Problems III

1. If there were a comet with an *aphelion* distance of 100 AU and a sidereal period of 408 years, what would be its
   a. average distance to the sun?

$a = 100$ AU, $T = 408$ yrs

$\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3}$ $\quad \frac{1^2}{1^3} = \frac{408^2}{R^3}$ $\quad R = 408^{2/3}$ $\quad \boxed{R = 55.01 \text{ AU}}$

   b. perihelion distance?

$a = R + c$ $\quad 100 = 55 + c$ $\quad c = 45$ AU $\quad p = R - c = 55 - 45$ $\quad \boxed{p = 10 \text{ AU}}$

   c. eccentricity?

$e = \frac{c}{R} = \frac{45}{55}$ $\quad \boxed{e = 0.82}$

   d. Synodic period?

$T = \frac{S}{S-1}$ $\quad 408 = \frac{S}{S-1}$ $\quad 408S - 408 = S$

$407S = 408$ $\quad S = 408/407$ $\quad \boxed{S = 1.002 \text{ yrs}}$

(b/c $R > 1$ AU)

2. Suppose you live on the strange planet of Kansas. One year on Kansas is exactly 500 days long. There is one other planet in this solar system, Utah, which takes 5500 days to go around the sun.
   a. What is the average distance from Utah to the sun? (Call 1 AU the semi-major axis of Kansas.)

$T_1 = 500$ days, $R_1 = 1$ AU, $T_2 = 5500$ days

$\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3}$ $\quad \frac{500^2}{1^3} = \frac{5500^2}{R^3}$

$R = \left(\frac{5500}{500}\right)^{2/3}$ $\quad \boxed{R = 4.95 \text{ AU}}$

   b. If Utah had an orbital eccentricity of 0.2, what would be its perihelion and aphelion distance?

$e = \frac{c}{R}$ $\quad c = (.2)(4.95)$ $\quad c = .989$

$p = R - c = 4.95 - .989$ $\quad \boxed{p = 3.96 \text{ AU}}$

$a = R + c = 4.95 + .989$ $\quad \boxed{a = 5.94 \text{ AU}}$

3. The average distance from the moon to the earth is 60 earth radii. It takes the moon 27.3 days to go around the earth once. The closest the moon ever gets to the earth is 56.7 earth radii.
   a. What is the farthest the moon gets from the earth?

$R = 60\ R_\oplus$, $p = 56.7\ R_\oplus$

$p = R - c$ $\quad 56.7 = 60 - c$ $\quad c = 3.3$

$a = R + c = 60 + 3.3$ $\quad \boxed{a = 63.3\ R_\oplus}$ ← earth radii

   b. What is the eccentricity of the moon's orbit around the earth?

$T = 27.3$ days

$e = \frac{c}{R} = \frac{3.3}{60}$ $\quad \boxed{e = 0.055}$

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4. How far from the earth would a satellite have to be in order to have a period of exactly 1 day? What would be interesting about the orbit if it was also "above" the equator?

$\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3}$ (T_1: moon, T_2: satellite)

$\frac{(27.3)^2}{(60)^3} = \frac{(1)^2}{R^3}$

$R^3 = 289.8$

$R = 6.62\ R_\oplus$

Since T = 1 day, it matches earth rotation! So always in same spot above the ground!

5. Imagine there is an asteroid orbiting the sun with an orbital eccentricity of 0.1. It also has a sidereal period of .617 years.

a. What would be the asteroid's perihelion distance?

e = 0.1

T = 0.617 yrs

$\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3}$ $\quad 1 = \frac{(.617)^2}{R^3}$ $\quad R = (.617)^{2/3}$ $\quad R = 0.725\ AU$

$e = c/R$ $\quad c = eR = (.1)(.725) = .0725$

$p = R - c$

$= .725 - .0725$

$p = 0.652\ AU$

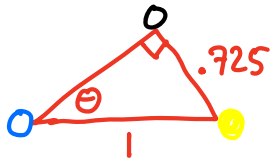
b. What would be the aphelion distance?

$a = R + c$

$= (.725) + (.0725)$

$a = 0.797\ AU$

c. What would be the angle of greatest elongation for this asteroid?



$\sin\theta = \frac{.725}{1}$ $\quad \theta = 46.5°$

6. Jupiter takes 11.9 years to go around the sun once. If it was a circular orbit, how fast would it be moving around the sun? Please find the speed in m/s. Note that 1 AU is 1.5×10^{11} meters.

$\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3}$ $\quad 1 = \frac{(11.9)^2}{R^3}$

$R^3 = (11.9)^{2/3}$

$R = 5.21\ AU$

$(11.9\ yrs)\left(\frac{365.25\ days}{yr}\right)\left(\frac{24\ h}{day}\right)\left(\frac{3600\ s}{h}\right)$

$= 3.91 \times 10^8$ seconds

$(5.21\ AU)\left(1.5 \times 10^{11}\ \frac{m}{AU}\right) = 7.82 \times 10^{11}\ m$

7. Assuming circular orbits, how does the speed of a planet going around the sun depend on its distance to the sun?

$\frac{T^2}{R^3} = K$ (some constant)

So $T^2 = KR^3$

Since $V = \frac{2\pi R}{T}$

$V^2 = \frac{4\pi^2 R^2}{T^2}$

So $V^2 = \frac{4\pi^2 R^2}{K R^3}$

$V^2 = \frac{4\pi^2}{K} \cdot \frac{1}{R}$

So $V \propto \sqrt{\frac{1}{R}}$

$V = \frac{2\pi R}{T} = \frac{2\pi (7.82 \times 10^{11})}{(3.91 \times 10^8)} = 12{,}600\ m/s$

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Answers

1. *a) R=55 AU b) perihelion=10 AU c) e=0.82 d) 1.002 yrs*
2. *a) R=4.95 AU b) perihelion=3.96 AU; aphelion=5.94 AU*
3. *a) apogee=63.3 R_e b) e=0.055*
4. *R=6.6 R_e; the satellite would appear motionless from the ground.*
5. *a) perihelion=0.653 AU b) aphelion=0.798 AU c) θ=46.5°*
6. *13,000 m/s*
7. $v \propto 1/\sqrt{R}$

$V = \frac{2\pi R}{T}$ $T^2 = R^3$

$v^2 = \frac{4\pi^2 R^2}{T^2}$

$v^2 = \frac{4\pi^2 R^2}{R^3}$